ABSTRACT

In literature a typical eigenvalue problem is commonly related with a linear differential equation. A basic example in the elementary theory of linear Partial Differential Equations (PDEs) concerns the eigenvalue problem for the Laplace operator on a bounded domain from the Euclidian space, subject to the homogeneous Dirichlet boundary condition. For this problem one can describe its entire spectrum as being a nondecreasing and unbounded sequence of positive real numbers. In the case of the nonlinear eigenvalue problems involving the $p$-Laplacian with $p$ a given constant in $(1,\infty)$ and $p\neq2$ the homogeneity of the problem enables us to consider that it still possesses the structure of a typical eigenvalue problem. However, the nonlinear character of the problem induces some complications in describing the set of all eigenvalues of the problem. It is known that the Ljusternik-Schnirelman theory ensures the existence of a nondecreasing and unbounded sequence of positive eigenvalues but in general this theory does not provide all eigenvalues. There are many other open questions concerning the set of eigenvalues of the nonlinear $p$-Laplacian. Next, we recall that even much less is known for the case of nontypical eigenvalue problems when the structure of the problem involves inhomogeneous differential operators. Finally, we point out that eigenvalue problems could also be regarded as a starting point in analyzing more complicated equations. Thus, alongside the study of the classical open questions from the topic (where we are aware of the fact that solving such kind of problems could be extremely difficult), in this project we propose the analysis of the following directions of research: the classification of isolated singularities for some PDEs; the analysis of a Rayleigh-type quotient corresponding to some eigenvalue problems involving rapidly growing differential operators; the study of some inhomogeneous torsional creep problems.